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# A study of the quenched $\bar{b}c$ mass spectrum at $\beta=6.2^*$

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We present an analysis of the mass spectrum of heavy quarkonia with non–degenerate quark masses. The heavier (bottom) valence quark is treated in a non–relativistic fashion and the other (charm) is a relativistic Wilson–like quark using the improved SW action. Such states provide and interesting probe between the relativistic B meson states and the non-relativistic bottomonium states.

#### 1. Introduction

The mass spectrum of quarkonia with valence bottom and charm quarks are interesting in a number of aspects. Though only a handful of events have been detected, upcoming B-factories should produce copious amounts of them. Hence, we are put in the interesting circumstance of making a pre-diction rather than a post-diction for non-exotic hadron masses. As they interpolate the non-relativistic bottomonium states and fully relativistic heavy-light states such as the B and  $B_s$  they provide an interesting probe of QCD dynamics. Of course, the calculation of these masses are also a first step in the calculation of decay constants and semi-leptonic widths, and these are of interest phenomenologically as an accurate calculation of  $f_{Bc}$  is crucial to obtaining as accurate as possible a result for  $f_B$  from LEP data [5] and the semi-leptonic decay  $B_c \to J/\psi l \overline{\nu}$  will give another estimate of the matrix element  $|V_{cb}|$ , which will be seen at the LHC-B [4].

#### 2. Lattice method

Previous quenched lattice calculations of the  $\overline{bc}$  spectrum [6,7] has been carried out at  $\beta = 5.7$ , where both valence quarks are simulated using the non-relativistic formalism [1]. On such

coarse lattices,  $am_b$  and  $am_c > 1$  and the effective Green's functions for both quark masses can be calculated without introducing large radiative corrections to the hamiltonian. However, it has been demonstrated that for such lattice spacings the charmonium spectrum is not reproduced very accurately. For quenched gluon fields at  $\beta = 6.2$ , quenched one can still use the non-relativistic approach for the b quarks and the relativistic formalism can now be used for treating the c quark. As one expects the dominant errors to be a function of  $am_c$ , which still approaches 1, we use the Sheikholeslami-Wohlert action [2], using a meanfield estimate of the clover coefficient  $c_{\rm sw}$ . While not eliminating all errors up to  $\mathcal{O}(a^2)$ , this mean field estimate approach does reduce errors significantly [8].

## 3. Computational Details

On a  $24^3 \times 48$  lattice at  $\beta = 6.2$ , corresponding to a lattice spacing of approximately 2.7 GeV, two point functions were constructed for the following states;  $(1,2)^1S_0$ ,  $(1,2)^3S_1$ ,  $^1P_1$  and  $^3P_{0,1,2}$  (using the notation  $(R)^{2s+1}L_J$ , where R is the radial excitation). The operators were calculated using Coulomb gauge fixed smeared sources[6]. In order to maximise the statistics, the non-relativistic Green's functions were calculated forwards and backwards in time, so that as many

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Table 1 Mass parameters used for states

κ	$am_b$	# Configs.
0.132	1.22	100
0.126	1.22	100
0.126	2.44	100

of the componants of the relativistic propagator could be used. In table 1 the quark mass parameters and the number of configurations are listed. We note that  $\kappa=0.126$  and  $am_b=1.22$  correspond roughly to charm and bottom quark masses from studies of charmonium and bottomonium systems respectively. The NRQCD hamiltonian was corrected to  $\mathcal{O}(mv^4)$ , using tadpole improved estimates of the relevant coefficients. As the configurations are quenched, the lattice spacing should be determined from one of the  $\bar{b}c$  observables. As the splitting  $\delta m_{P,S}={}^1P_1-{}^1\overline{S}$  for charmonium and bottomonium only changes by a few MeV, we took an average of these splittings and hence assumed that  $\delta m_{P,S}(\bar{b}c)=0.4545~\mathrm{GeV}$ .

#### 3.1. Mixing

One expects that the  ${}^{1}P_{1}$  and  ${}^{3}P_{1}$  states to mix (this has been observed at  $\beta = 5.7$  [6] and with two relativistic propagators at higher  $\beta$ 's with different masses around that of charm). Therefore we calculated the following matrix of correlators

$$\begin{pmatrix} <^{1}P_{1}^{1}P_{1}^{\dagger}> & <^{1}P_{1}^{3}P_{1}^{\dagger}> \\ <^{3}P_{1}^{1}P_{1}^{\dagger}> & <^{3}P_{1}^{3}P_{1}^{\dagger}> \end{pmatrix} .$$

However, for all three combinations of quark masses, the off-diagonal elements of this matrix were very noisy and were consistent with zero.

#### 4. Results

In figure 1 the results for the  $\overline{bc}$  spectrum using the the best estimates for the bottom and charm quark masses are compared with the previous lattice calculation at  $\beta = 5.7$  [6] and a potential model calculation [3]. The R=1 states at both  $\beta$ 's are consistent with each other. For the S-wave states, where we have a measurement of the R=2 states, the agreement is to within two standard deviations, albeit with large errors.

The potential model calculation agrees with the lattice results except for the hyperfine splitting. It is not clear whether this difference is due any systematic error in the potential model or by a choice of the scale using  $\delta m_{P,S}$ .

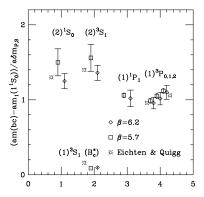
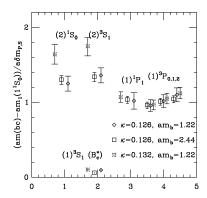
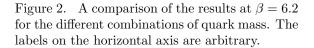


Figure 1. A comparison of the results at  $\beta = 5.7[6]$ ,  $\beta = 6.2$  ( $\kappa = 0.126$ ,  $am_b = 1.22$ ) and a potential model calculation. The labels on the horizontal axis are arbitrary.

In figure 2 the results for the  $\bar{b}c$  spectrum for different combinations of quark masses at  $\beta=6.2$  are shown. Once again, there is a general broad agreement. For the splittings, we find that the splittings behave as expected. For example, in figure 3 we see that as the b quark mass is increased, and the system behaves more like a heavy-light bd state the splitting tends to zero. Likewise, the fine splitting (in figure 4) is constant, indicating that  $\delta m_{P,S}$  should be insensitive to variations in the quark masses.





### 5. Conclusions

It is very promising that the quenched lattice results at two different lattice spacings are in good agreement with each other, despite potential concerns over the size of the discretisation error at  $\beta=5.7$ . The broad agreement with potential models is also quite interesting as it is not clear that a semi-relativistic system such as this could be modelled with a simple potential. Given the amount of research that has been done on the  $\overline{b}b$  and  $\overline{b}(u,d)$  spectrum, it would be interesting to see how the splittings of these states vary as the masses of the quarks are changed.

The absence of mixing between the  ${}^{3}P_{1}$  and  ${}^{1}P_{1}$  states at  $\beta=6.2$  is quite strange, given its presence at  $\beta=5.7$ . It is of course possible that the mixing is a strong function of the lattice spacing and that the mixing is very small in the continuum. This would give a qualitatively different result from potential models. In order to verify if this is true, the calculation should be repeated using some other set of operators, defined in a different gauge or a gauge invariant fashion.

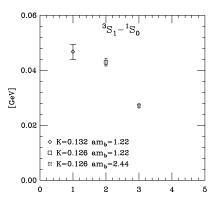


Figure 3. A comparison of the  ${}^3S_1 - {}^1S_0$  splitting at  $\beta = 6.2$  for the different combinations of quark mass. The labels on the horizontal axis are arbitrary.

#### 6. Acknowledgements

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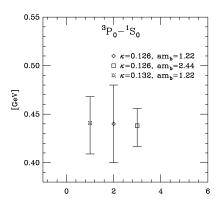


Figure 4. A comparison of the  $^3P_0-^1S_0$  splitting at  $\beta=6.2$  for the different combinations of quark mass. The labels on the horizontal axis are arbitrary.